GCSE Maths - Algebra

Numerical Iteration (Higher Only)

Worksheet

NOTES

SOLUTIONS

This worksheet will show you how to work out different types of numerical iteration questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Using a starting value of $x_0 = 9.3$, use numerical iteration to find the solution to the equation $x^2 - 10x + 6 = 0$ to 3 decimal places.

Step 1: Rearrange the equation so that it is in the correct form.

$$x^2 - 10x + 6 = 0$$
$$x^2 = 10x - 6$$
$$x = \sqrt{10x - 6}$$

Step 2: Add in the iteration notation.

$$x_{n+1} = \sqrt{10x_n - 6}$$

Step 3: Substitute in the starting value (x_0) for x_n to obtain x_1. Repeat until the same answer is obtained twice.

$$x_1 = \sqrt{10 \times 9.3 - 6} = 9.327 \ldots$$
$$x_2 = \sqrt{10 \times 9.327 - 6} = 9.342 \ldots$$
$$x_3 = \sqrt{10 \times 9.342 - 6} = 9.350 \ldots$$
$$x_4 = \sqrt{10 \times 9.350 - 6} = 9.354 \ldots$$
$$x_5 = \sqrt{10 \times 9.354 - 6} = 9.356 \ldots$$
$$x_6 = \sqrt{10 \times 9.356 - 6} = 9.357 \ldots$$
$$x_7 = \sqrt{10 \times 9.357 - 6} = 9.358 \ldots$$
$$x_8 = \sqrt{10 \times 9.358 - 6} = 9.358 \ldots$$

Now that we have the same answer twice (to 3 decimal places), this is our final solution.

$$x = 9.358$$

Guided Example

Work out the solution to $x^3 - 15x + 12 = 0$ using numerical iteration, beginning with $x_0 = 3.3$. Give the solution to 3 decimal places.

Step 1: Rearrange the equation so that it is in the correct form.

Step 2: Add in the iteration notation.

Step 3: Substitute in the starting value (x_0) for x_n to obtain x_1. Repeat until the same answer is obtained twice.

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

1. Calculate the solutions to the following, using numerical iteration. Give the solutions to 3 decimal places.

a) $x^2 + 3x - 80 = 0$, starting with $x_0 = 7.6$

b) $2x^3 - 8x^2 - 5 = 0$, with a starting value of $x_0 = 4.1$

c) $2x^3 + 4x = 14$, with a starting value of $x_0 = 1$

d) $0.5x^3 + 2.5x - 10 = 0$, with a starting value of $x_0 = 2$